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Axion Hilltops, Kahler Modulus Quintessence and the Swampland Criteria

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Abstract: We study the interplay between extrema of axion potentials, Kahler moduli stabilization and the swampland criteria. We argue that moving away from the minima of non-perturbatively generated axion potentials can lead to a runaway behavior of moduli that govern the couplings in the effective field theory. The proper inclusion of these degrees of freedom resolves the conflict between periodic axion potentials and the gradient de Sitter criterion, without the need to invoke the refined de Sitter criterion. We investigate the possibility of including this runaway direction as a model of quintessence that satisfies the swampland criteria and find phenomenologically viable but unstable trajectories that require fine-tuning of initial conditions for the axion. The attempt to stabilize these trajectories along the axionic directions, while respecting the swampland criteria, leads the ground state of the model outside the regime of validity of our approximations and we conjecture that the class of models with such stabilized runaway valleys lies in the swampland.

Contents

1. Introduction

Axions are ubiquitous in theoretical physics. The Peccei-Quinn mechanism [1], a proposed resolution to the strong CP problem, promotes the QCD θ-angle to a dynamical axion field. Multi-axion models are also commonly used for cosmological model building, for example natural inflation [2] models, axion quintessence models [3] etc. Axions are also omnipresent in string theory compactifications. In type II strings they arise from the Ramond-Ramond p-form potentials wrapping internal cycles of the compactification manifold and in the case of SUSY compactifications, give the imaginary parts of the chiral multiplets in the 4D effective theory [4].

In this paper we explore various aspects of axion models that originate in string theory and their interplay with the swampland criteria [5] that received much attention over the past few months (e.g. [9, 10, 11, 18, 21, 22, 23, 25] among many others). Our particular focus will be on the interplay with what we'll refer to as the "distance" and "gradient" criteria. The former states that field values must remain within a planck unit of the ground state of our effective field theory, while the latter states that the scalar potential must obey

$$\nabla V \geq cV \qquad c = \mathcal{O}(1). \tag{1.1}$$

The latter criterion has been refined, as proposed in [6] and further justified in [7] (see also [8]), to include a condition on the second derivative. This refined criterion allows for violations of the gradient criterion, provided the Hessian of the potential has a sufficiently negative eigenvalue. In this paper we will avoid making use of the refined criterion and instead look for ways to respect the gradient criterion.

In section 2 we point out that the hilltops of axion potentials are not in conflict with the gradient de Sitter swampland criterion [5]. The reason is that the same non-perturbative effects that generate the axion potentials also generate a potential for the moduli that govern the coupling and the total potential satisfies the gradient criterion. We do not touch the similar apparent conflict in the case of the Higgs potential [9, 10] as the embeddings of the Higgs potential into a string theoretical setup are less clear.

In section 3 we attempt to harness this non-perturbatively generated potential to construct a quintessence model. This model uses the real part of a Kahler modulus at an axionic hilltop as a quintessence field. We forego the use of any supersymmetry breaking uplift ingredient such as anti-D3 branes. The model in principle contains trajectories that can account for a small, nearly constant, positive energy density of the universe that persists for a Hubble time, but requires fine tuning of initial conditions. We attempt to remedy this by considering additional non-perturbative effects in a racetrack scenario, but find that for a choice of parameters that doesn't violate the gradient and distance swampland criteria, the ground state of the theory lies outside the regime of validity of the approximations used. We conjecture that models of this type lie in the swampland. We conclude with a review of our results as well as some general comments and speculations. [1]

2. Peccei-Quinn in String Theory

It was recently suggested [9] that the Peccei-Quinn mechanism for solving the strong CP problem is in tension with the gradient criterion (1.1).

The authors of [9] consider a simple model of an action with a potential consisting of a small slowly varying quintessence term and the usual cosine potential for the QCD axion field, with parameters within the bounds dictated by observation. They then observed that at the local maximum of the cosine potential the swampland criterion (1.1) is violated by several orders of magnitude. A modified criterion has been proposed in [7], which among other things removes this conflict, however we will argue that the old criterion is also not violated if the theory is properly embedded in string theory.

Here we analyze realizations of this scenario in string theory and show how this problem is averted. The key point is that the gauge coupling is itself a dynamical degree of freedom in string theory and the same non-perturbative effects that generate the axion potential also couple the potential to the gauge coupling.

[1] As this work was being completed papers involving a similar set of ingredients appeared [21, 22, 23]. The overlap with [22] concerns discussions of transplanckian axions, [23] also considers using the size modulus as the quintessence field but without axion dynamics and no tree level superpotential. [21] contains a useful analysis of the constraints on initial conditions near hilltops, which we incorporated in section 3.

The Peccei-Quinn mechanism [1] for the resolution of the strong CP problem instructs us to promote the QCD θ-angle to a dynamical degree of freedom a. This degree of freedom naturally couples to the Yang-Mills instanton density $F \wedge F$ and the effect of the instantons generates an effective potential for a. This potential can be approximated by a dilute instanton gas calculation [12]. The theory has approximate saddles corresponding to n instantons and $\bar{n}$ anti-instantons, all widely separated. Evaluating the contribution of these saddles to the path integral gives

$$\sum_{n,\bar{n}} \frac{1}{n!\bar{n}!} \left(Ke^{-S_0}\right)^{n+\bar{n}} e^{i(n-\bar{n})a} \mathcal{V}^{n+\bar{n}} = \\ exp(2K\mathcal{V}e^{-S_0}\mathrm{cos}a), \tag{2.1}$$

where S_0 is the single-instanton action, K is a one-loop determinant around a single instanton and $\mathcal{V}$ is the volume of the moduli space of a single instanton. This volume is proportional to the volume of the spacetime, since there is a zero-mode corresponding to the location of the instanton, however in the presence of internal symmetries (e.g. R-symmetry) there are additional moduli that need to be integrated over, giving additional multiplicative factors.

More importantly, the action of the instanton itself depends on the value of other moduli, specifically the dilaton and the volumes of internal cycles.

Let's consider some specific examples. We start with a stack of D3 branes in a type IIB compactification. The effective 4D theory will of course contain the $\mathcal{N}=4$ SYM action coming from the worldvolume of the branes, but since the internal space is compact it will also include terms governing the dynamics of all the moduli fields that aren't stabilized at energies above the cutoff. The worldvolume action for the branes contains a coupling to the RR axion:

$$\int d^4x C_0 F \wedge F \tag{2.2}$$

so C_0 is precisely the θ-angle in the $\mathcal{N}=4$ SYM, but in the full theory it is in fact a dynamical field. We can now compute the non-perturbative potential induced by the instantons.

The instanton action is proportional to the inverse string coupling

$$S_0 \propto \frac{1}{g_{YM}^2} = \frac{1}{g_s} = e^{-\phi} \tag{2.3}$$

In fact we can recognize the Yang-Mills instantons as D-instantons dissolved inside the branes. In the absence of warping, there are also contributions from D-instantons located at any point in the internal space that couple to C_0 in exactly the same way. The full non-perturbative potential for C_0 will then include an integral over the whole moduli space of these D-instantons giving an overall factor of the internal volume. The leading order non-perturbative contribution to the potential coming from the instantons is

$$V \propto -Be^{-S_0}\cos a = -B\exp(\alpha e^{-\phi})\cos a, \tag{2.4}$$

with $\alpha = \mathcal{O}(1)$ and B proportional to the volume of the instanton moduli space as well as the one-loop determinant around a single instanton.

We now see that the potential conflict with the swampland criterion (1.1) due to the local maximum at $a = \pi$ is alleviated for small values of ϕ, i.e. at weak coupling. If we fix the axion to be at the local maximum of the cosine, the potential still has a dependence on the dilaton, which satisfies the swampland criterion at weak coupling.

We can consider alternative realizations of axions. For example we can consider higher dimensional branes wrapping internal cycles. Consider a D5 wrapping a 2-cycle. Then similarly to the D3 case, a CP violating term arises from part of the Chern-Simons action

$$\int d^4x d^2y C_2 \wedge F \wedge F \tag{2.5}$$

Here the axion is given by

$$a = \int d^2y C_2 \tag{2.6}$$

Again, we recognize the worldvolume instantons as being dissolved D1-brane instantons. The action of these instantons is proportional to $e^{-\phi}\int B_2$ over the 2-cycle. This is again a modulus and the instanton contribution results in a coupling to this modulus of the form (2.4) with $e^{-\phi}$ replaced by $\int B_2$ over the wrapped cycle.

Similarly, we can consider D7 branes wrapping a 4-cycle. The axion will then be given by an integral of C_4 over that 4-cycle and the gauge instantons are dissolved Euclidean D3-instantons. The Kahler modulus that couples to the axion potential controls the size of this 4-cycle.

Finally we can also consider heterotic string theory. Here the coupling to the internal volume is manifest, as all the fields are spacetime fields and must be integrated over the full internal manifold when going to the 4D description.

Of course all of these scenarios can be related to each other by dualities, so it's not surprising that they lead us to the same conclusion. The common point illustrated in these examples is that upon coupling the field theory to gravity by embedding it in string theory, the gauge coupling becomes a dynamical variable itself. The instanton contributions don't just provide a potential for the axion, but couple the axion to a volume modulus and/or dilaton and the would-be local maximum in the axion potential still has a non-zero gradient along this direction satisfying the bound.

The examples considered above are all supersymmetric and the Kahler and axion moduli are part of the same chiral multiplet in the effective 4D theory. The whole multiplet is stabilized and so the masses of the Kahler modulus and the axion are roughly the same. Indeed this is a problem one has to overcome when trying to obtain a QCD axion from string theory, as pointed out in [13]. In a realistic model we want to eventually break supersymmetry anyway, so one may hope to use this breaking to avoid this problem. When supersymmetry is broken, it's possible to give each of these fields different masses and one might worry that this also breaks the above argument. Specifically one can imagine stabilizing the Kahler modulus at much higher energies than the axion and claim that the gauge coupling is effectively constant for all values of the axion, if we study the theory at low enough energies so that the Kahler modulus fluctuations are integrated out, and the effective theory only consists of the axion with a cosine potential. However, such reasoning is only valid in the absence of cross-terms between the heavy and light fields, and (2.4) is precisely such a cross-term.

The idea of bypassing the gradient criterion by making the gauge coupling dynamical was already considered by the authors of [9]. One of their proposals was to couple the quintessence field to the gluons, thus effectively making the gauge coupling depend on the quintessence field, and was rejected as a viable option for the QCD axion, since this would result in a coupling between the quintessence field and the nucleons, in tension with fifth-force constraints between nucleons.

The situation we describe is similar in form, but differs in that the gluons don't couple to a separate light quintessence field. Instead the gluons are coupled to a much heavier field (the modulus that governs the coupling) that is not the same as the quintessence field considered in [9]. Upon moving to the axion hilltop this modulus remains heavy, which suppresses the fifth-force interactions between nucleons, but also develops a runaway direction that respects the gradient criterion.

One may worry that we are ignoring other contributions to the potential which in fact stabilize the Kahler moduli so that the total potential still violates the gradient criterion. However the instanton contribution considered above is precisely such a contribution, and flipping its sign will always convert a minimum to a runaway. The only way for this sign change to not affect the stability of the Kahler modulus is if it's stabilized by other more dominant effects to begin with.

3. Kahler quintessence at axion hilltop?

In the previous section we found that deviating far from the minimum of the axion potential leads to an exponential potential for the coupling modulus. Such exponential potentials can be the candidates for a quintessence model that satisfies the swampland criteria. The Kahler modulus corresponding to the coupling constant which is part of the same multiplet as the QCD axion is not a good candidate as a quintessence field. On the other hand, we can consider other axions that don't couple strongly to the standard model sector, and their corresponding Kahler moduli can potentially serve as quintessence fields. In this section we will attempt to harness these dynamics to construct such a toy model of quintessence. We will find that the hilltop of a (non-QCD) axion potential provides us with an unstable but otherwise viable trajectory in field space to realize quintessence dynamics. We will also describe an attempt to stabilize this trajectory by introducing additional non-perturbative effects to create a dip in the axionic potential. This will turn out to be more difficult than it seems without running into tensions with either the gradient criterion or the distance criterion.

3.1 Surveying the hilltop

First we review the KKLT moduli stabilization procedure [14]. Kahler moduli stabilization in KKLT-like scenarios is achieved by considering the non-perturbative brane-instanton and gaugino condensation effects. The resulting potential contains competing exponential terms with different exponents, that balance each other out at some intermediate values of the volume. We take the effective superpotential and Kahler potential for a single Kahler modulus to have the following general form:

$$\begin{aligned} W &= W_0 + A\exp(-aT) \\ e^K &= \frac{1}{X^3} \end{aligned} \tag{3.1}$$

W_0 is independent of the Kahler modulus $T = X + iY$, where X could be the volume of some 4-cycle in the internal geometry, Y is the axion corresponding to the 4-form threading that cycle. a is of order one and generally depends on the other moduli as does A, which is a one-loop determinant that incorporates an integral over any instanton moduli other than spacetime translation modes.

In order to be able to ignore the higher order α' corrections X needs to be sufficiently large and we must check that this is the case at the end of our calculations.

To obtain the potential we need to compute

$$\begin{aligned} D_T W &= -\frac{3}{2X}W_0 - (a + \frac{3}{2T})A\exp(-aT), \\ K^{T\bar{T}} &= \frac{4}{3}X^2 \end{aligned} \tag{3.2}$$

This results in the following potential (fig. 1).

$$V = \left(\frac{4}{3}\frac{a^2}{X} + \frac{4a}{X^2}\right)A^2 e^{-2aX} + \frac{4a}{X^2}W_0 A e^{-aX}\cos[aY] \tag{3.3}$$

For a sypersymmetric minimum we need

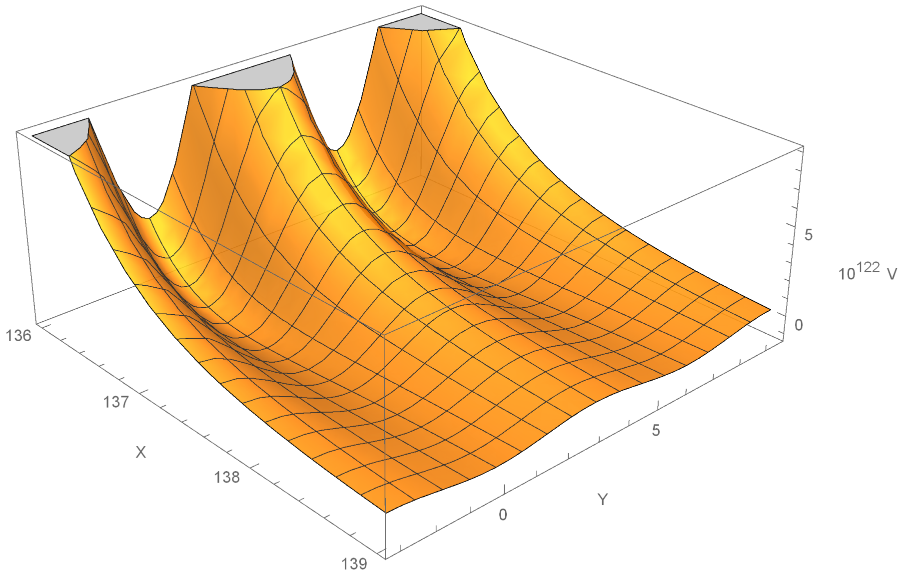

Figure 1: The full scalar potential. The valleys contain the SUSY minimum, while the ridges are unstable.

$$DW = 0 \implies W_0 = -(\frac{2}{3}aX + 1)Ae^{-aT}, \tag{3.4}$$

which will stabilize the axion at zero and the volume at some finite value. We will need this value to be large enough, so that the expansions in T^{-1} make sense, but no so large that the KK modes become lighter than the Kahler modulus. At this minimum of the potential we have

$$V_c = -\frac{4}{3}\frac{a^2}{X_c}A^2 \exp(-2aX_c) \tag{3.5}$$

The energy at the minimum is negative. At this point KKLT-type constructions invoke an uplift mechanism to obtain a meta-stable de Sitter minimum. We will take a different path and look for hilltops in the axion potential.

We consider setting the axion to $aY = \pi$. This will reverse the sign of the cosine term in (3.3). We have

$$V(X + i\pi/a) = \left(\frac{4}{3}\frac{a^2}{X} + \frac{4a}{X^2}\right) A^2 e^{-2aX} - \frac{2a}{X^2} W_0 A e^{-aX} \tag{3.6}$$

Where we note that $W_0 A$ is negative, so the minus sign in front of the last term is misleading. The value of the potential is in fact positive definite and pushes the system to larger cycle volumes. This unstable configuration should be viewed as a non-equilibrium state in the same effective theory as the SUSY minimum. To ensure this, we will require that the canonically normalized modulus, $\sqrt{\frac{3}{2}}\log T$, doesn't differ from $\sqrt{\frac{3}{2}}\log T_c$ by more than $\mathcal{O}(1)$ in Planck units to satisfy the distance criterion, in other words the volume can't change by more than an order of magnitude.

If we set $X = X_c$, but keep the axion at the hilltop the potential equals

$$V(X_c + i\pi/a) = \left(\frac{8a}{X_c^2} + \frac{4a^2}{X_c}\right) A^2 e^{-2aX_c} \tag{3.7}$$

We can check whether the potential (3.6) satisfies the swampland criterion within this region. We find that

$$\frac{|\partial_X V|}{V} \sim a + \mathcal{O}(\frac{1}{X}) \tag{3.8}$$

Satisfying the swampland de Sitter criterion requires $a \gtrsim \mathcal{O}(1)$. This simultaneously guarantees that the axion hilltop lies within a Planck distance of the minimum. The value of the potential at $X_c + i\pi/a$ can then be tuned by tuning W_0, i.e. by choosing appropriate internal fluxes.

For our model if we take $a \sim \mathcal{O}(1)$, and we want $V \sim 10^{-122}$, we need $W_0 \sim 10^{-58}$, which leads to $X_c \sim 137$.

If we start near $T = X_c + i\pi/a$, we expect the dynamics to initially follow a slow roll toward larger X. However, the axion will tend to deviate from its hilltop value and roll down to zero. At this point the potential for X will instead cause it to decrease down to the SUSY minimum. The rolling down of the axion will be accompanied by a transition from a positive energy density to a negative energy density. The duration of this process mainly depends on how quickly the axion leaves the hilltop and is therefore highly sensitive to initial conditions (fig 2). The phenomenological viability of this model involves several considerations.

The first set of considerations comes from the requirement that the quintessence field is sufficiently "dark" to avoid tension with fifth-force constraints. Ensuring that our Kahler modulus doesn't couple to the Stanard Model fields is subtle [15]. The usual assumption is that we can insert the Standard Model into our compactification in such a way that the cycle controlled by our Kahler modulus doesn't couple to it. However, there is generically kinetic mixing between the various fields so this decoupling must be checked within specific models on a case-by-case basis.

The second set of phenomenological considerations concerns the tuning of initial conditions. On the one hand, if the axion manages to stay at the hilltop for long enough, X will roll to transplanckian distances. We need to make sure that this does not happen within a Hubble time. This constraint is not very strong. As pointed out earlier, a transplanckian deviation of the canonically normalized field $\log X$, means an order of magnitude change in X, which translates to a double exponential change in the value of the potential itself. Simply requiring that the model is consistent with the observational constraints on the variation of the cosmological constant will already place the model within the bounds of the distance criterion.

On the other hand, starting too close to the hilltop can lead to formation of domain walls, as different regions of space can roll off in different directions. The presence of such domain walls is problematic because they could be unstable, decompose and give rise to other axions. As pointed out in [21], we must start sufficiently far from the hilltop so that quantum fluctuations don't lead to this domain wall formation, but sufficiently close so that we don't roll off along the axion directions too quickly. In [21] it was shown that the lower bound on the deviation from the hilltop is easy to obey and still leaves a range of initial conditions for a phenomenologically viable dark energy models.

While suitable initial conditions exist, as with all "hilltop" models, there is a problem in justifying these initial conditions as the dynamics of the system naturally lead it away from them. One option would be to tunnel to them from a local minimum of similar energy, however demanding that the model respects the swampland criteria excludes this option. A more natural situation would be if the model could be modified so that this runaway trajectory was stabilized along the axionic direction. In the next section, we will attempt to construct exactly such a model. We will ultimately fail, but in an instructive way.

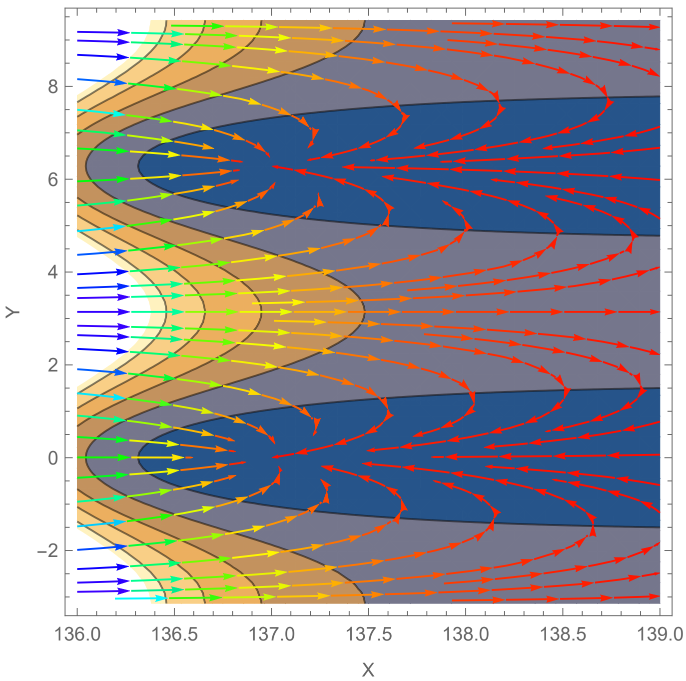

Figure 2: Contour plot of the potential with the gradient streamlines schematically representing the evolution of the system in a slow-roll approximation.

3.2 Landscaping our way into the swamp

To bypass the initial conditions problem, we can attempt to modify the model to create a local minimum along the axionic direction at positive energy. In general, the number of non-perturbative effects in a given compactification will exceed the number of axion fields, and so the potential will generally admit local minima along the axionic directions. The authors of [16] have devised a powerful framework for systematically analyzing the potential for a large numbers of axions with non-perturbative cosine potentials and local meta-stable minima are a generic feature of such axionic potentials. In these works they made no attempt to embed the axions in string theory so they had no reason to consider a dynamical coupling. As such, these potentials would seem to violate the gradient de Sitter swampland criterion in the same way that the Peccei-Quinn mechanism did, but are consistent with the refined version. Moreover, as we argued in section 2, it's impossible to ignore the modulus that governs the coupling far from the global minimum of the axion potential and so we obtain an exponential potential for the coupling that generically satisfies the gradient criterion.

If such a local minimum happens at positive energy, it would create a valley with exponential behavior along the X direction, providing a stable trajectory for a quintessence model.[2] The walls of this valley could either then shallow out at sufficiently large X allowing the axion Y to roll back down to its global minimum, or the system could tunnel out of the valley and roll back to the SUSY minimum. This way the dynamics would naturally justify initial conditions away from the global minimum, have a sufficiently long-lasting slow-roll period at exponentially suppressed vacuum energy and end in an eventual decay into the SUSY AdS minimum. The dynamics would never leave the regime of validity of the EFT describing the SUSY minimum.

To obtain local minima in the axion potential we need more non-perturbative contributions than axions. The simplest realization of this is the well-studied "race-track" scenario [17], where the Kahler potential and superpotential are given by

$$\begin{aligned} W &= W_0 + A\exp(-aT) + B\exp(-bT) \\ e^K &= \frac{1}{X^3} \end{aligned} \tag{3.9}$$

The resulting potential is

[2][15] considers models with similar runaway "valleys" and point out problems related to strong quantum corrections to the potential after supersymmetry breaking. Since our EFT is ultimately supersymmetric, we can expect to have control over the quantum corrections to the scalar potential, avoiding these problems. However we will encounter other difficulties related to the α' corrections to the Kahler potential.

$$V = \frac{e^{-(a+b)X}}{6X^2} \quad \Big(aA^2(aX+3)e^{(b-a)X} + bB^2(bX+3)e^{(a-b)X}$$
$$+\ AB(2abX+3a+3b)\cos[(a-b)Y]$$
$$+\ 3W_0(aAe^{bX}\cos[aY] + bBe^{aX}\cos[bY])\Big) \qquad (3.10)$$

In order to satisfy the gradient de Sitter criterion at axionic maxima we require $a, b \gtrsim \mathcal{O}(1)$. Choosing these constants to satisfy this condition will also land us on the safe side of the distance criterion, which requires at least one of the following:

$$aX \gtrsim 1 \qquad bX \gtrsim 1 \qquad (a-b)X \gtrsim 1, \qquad (3.11)$$

to hold, depending on which cosine term is dominant,[3] otherwise neighboring local axionic minima will be more than a Planck distance away from the absolute minimum and we can no longer trust the effective field theory at one minimum to describe the other. This would also constitute a violation of the weak gravity conjecture as described in [18].

The absolute minimum of the potential $T = X_c + 0i$ can once again be found by demanding $DW = 0$, which leads to

$$W_0 = -(1 + \frac{2}{3}aX_c)e^{-aX_c} - (1 + \frac{2}{3}bX_c)e^{-bX_c} \qquad (3.12)$$

To stay within the regime of validity of the same EFT that describes this minimum, X needs to remain within an order of magnitude of X_c. If we fix $X = X_c$ and study the potential along the axionic directions we obtain

$$V = \frac{e^{-(a+b)X_c}}{6X_c^2} \quad \big(aA^2(aX_c+3)e^{(b-a)X_c} + bB^2(bX_c+3)e^{(a-b)X_c}$$
$$+\ AB(2abX_c+3a+3b)\cos[(a-b)Y]$$
$$-\ (aA^2(2aX_c+3)e^{(b-a)X_c} + aAB(2bX_c+3))\cos[aY]$$
$$-\ (bB^2(2bX_c+3)e^{(a-b)X_c} + bAB(2aX_c+3))\cos[bY]\ \big) \qquad (3.13)$$

In principle we have enough freedom in the parameters of the above potential to set the coefficients of any of the cosines as well as the Y-independent term to anything we want. It is therefore not too difficult to construct a potential that would have a local minimum along the Y-direction near $X = X_c$. Let us rewrite the above potential as

[3] This is essentially the requirement that the decay constant of the dominant cosine term in the axion potential axion potential is subplanckian. The reason X appears in the condition is that the distance conjecture holds for canonically normalized fields, which Y is not.

$$V = k\left(\mathcal{E} + \cos[(a-b)Y] - \mathcal{A}\cos[aY] - \mathcal{B}\cos[bY]\right) \tag{3.14}$$

$$\begin{aligned}
k &= \frac{e^{-(a+b)X_c}}{6X_c^2} AB(2abX_c + 3a + 3b) \\
\mathcal{E} &= \frac{aA^2(aX_c+3)e^{(b-a)X_c} + bB^2(bX_c+3)e^{(a-b)X_c}}{AB(2abX_c+3a+3b)} \\
\mathcal{A} &= \frac{aA^2(2aX_c+3)e^{(b-a)X_c} + aAB(2bX_c+3)}{AB(2abX_c+3a+3b)} \\
\mathcal{B} &= \frac{bB^2(2bX_c+3)e^{(a-b)X_c} + bAB(2aX_c+3)}{AB(2abX_c+3a+3b)}.
\end{aligned} \tag{3.15}$$

Our task now consists of finding appropriate values of $\mathcal{E}, \mathcal{A}, \mathcal{B}, a, b$ and we could then in principle solve (3.15) to recover A, B, X_c as well as W_0 via (3.12).

A simple example of parameters that yields a local minimum is

$$a = 1 \qquad b = 0.5 \qquad \mathcal{A} = 1 \qquad \mathcal{B} = 3 \tag{3.16}$$

and the value of $\mathcal{E}$ doesn't really affect the existence of the minimum, only its height. Plotting the scalar potential with this choice of parameters (figs 3 and 4) we clearly get the valley that we seek.

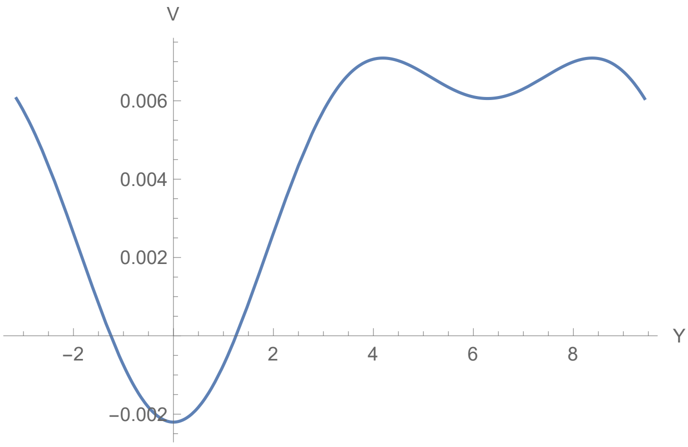

Figure 3: The axion potential at $X = X_c$ with parameters chosen to give a local minimum.

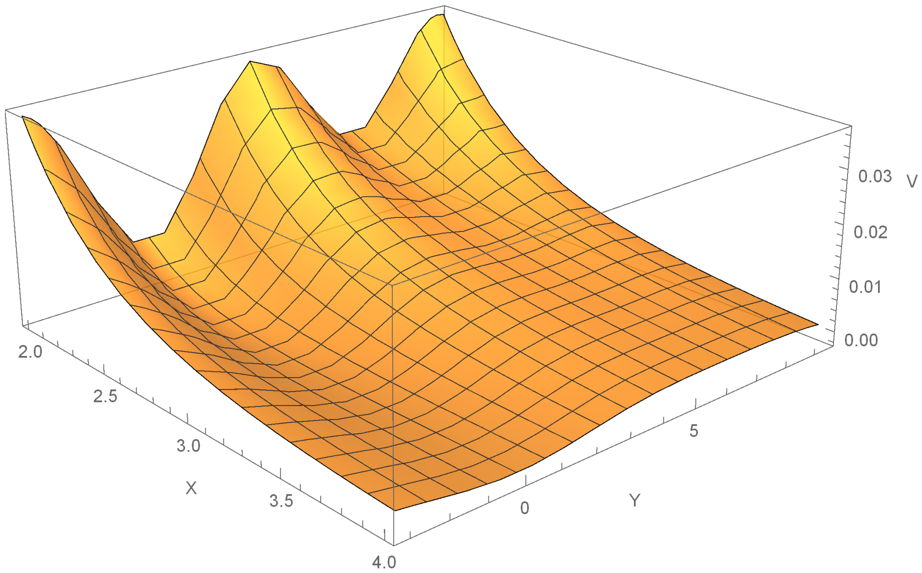

Figure 4: The full scalar potential with two non-perturbative effects. The deep valley contains the SUSY minimum, and the shallow valley provides an exponential slope.

This choice, however, is physically unacceptable.

Solving for X_c we obtain $X_c = 3$ which is too small to be able to ignore α' corrections to the Kahler potential. Moreover, including these corrections typically leads to even larger compactification volumes, so this choice of parameters is simply inconsistent with the dynamics of the model.

The source of our troubles can be seen by examining (3.15). Note that $\mathcal{E}$ is always less than $\mathcal{A}$ and $\mathcal{B}$. Local minima will happen when some of the cosine terms in the potential end up with opposite phases. If we wish to generate a local minimum in the cosine terms that gets lifted to positive energy by $\mathcal{E}$, we require that the higher frequency oscillations have smaller amplitude, but higher second derivative so that the extremum is a minimum, i.e. if $a > b$ then we need roughly

$$\begin{aligned} \mathcal{A} &< \mathcal{B} \\ a^2\mathcal{A} &> b^2\mathcal{B} \end{aligned} \tag{3.17}$$

However, to ignore α' corrections, we also require $X_c \gg 1$ and for $a > b$ this forces $\mathcal{B} \gg \mathcal{A} \sim \mathcal{O}(1)$.[4] This means that $\frac{a}{b}$ must be exponentially large, which in turn implies that there must be an exponential hierarchy between the actions of the two

[4]Note that this means that the $\cos[(a-b)Y]$ term in (3.14) is never dominant, which steers us clear

non-perturbative effects in our model, i.e. the action of one type of non-perturbative effect is smaller than even the perturbative corrections to the other non-perturbative effect. This puts the model outside the regime of validity of its own approximations and therefore into the swampland.

One may wonder if better results can be achieved by including additional axions with more non-perturbative effects along the lines of [19]. This only makes it more difficult to achieve positive energy local minima in the axion directions. Indeed, in the second paper of [16] the authors determine the maximum energy for local minima in generic multi-axion landscapes and find that it goes down with the number of axions. More non-perturbative effects means more exponential hierarchies in the amplitudes of the oscillatory terms, which can the only be compensated by choosing extreme values of the Kahler moduli or the decay constants, placing the model in the swampland.

4. Conclusion

The swampland criteria aim to articulate restrictions on effective field theories that can allow one to determine from a dimensionally reduced perspective whether a given model can arise from a string theory compactification. In this work we have examined the interplay between the gradient de Sitter criterion, the distance criterion and the non-perturbative axion potentials that generically arise in string compactifications. First we pointed out that the tensions between local maxima of axion potentials and the gradient criterion discussed in [9] are in fact not there if one properly considers the coupling parameter as a dynamical variable. Moving to hilltops in the axion potential destabilizes the real parts of the Kahler moduli and the potential has exponential behavior in that direction. If moving to the hilltop doesn't violate the field excursion criterion, then the exponential behavior will also satisfy the gradient criterion.

Exponential potentials are viable ingredients for quintessence models, and in the rest of the paper we explore the possibility of constructing such a model within the KKLT moduli stabilization framework. Rather than using a supersymmetry-breaking uplifting ingredient, such as anti-branes, we propose to view the current state of the universe as a non-equilibrium, non-supersymmetric positive energy state in the same supersymmetric effective field theory as the KKLT AdS minimum. In this non-equilibrium state the axion is near its hilltop and the destabilized Kahler modulus acts as a quintessence field.

We discussed the space of parameters and initial conditions that could ensure a suitably long period of exponentially small, nearly constant positive energy density, without violating the distance criterion. While suitable paths through configuration

of the problems described in [18], related to enhancement of the decay constant to transplanckian values.

space exist, they require fine tuning of parameters and initial conditions. We attempted to remedy the initial conditions problem by turning the axionic hilltop into a local valley stabilizing the desired trajectory and turning it into a local attractor, by considering a "racetrack" scenario with two non-perturbative contributions to the superpotential and found that generating the desired valley was incompatible with the swampland criteria. We argued that this is a generic problem for this kind of model and that adding more fields and non-perturbative effects will only make creation of such valleys more difficult. It is interesting that although this valley satisfies the gradient criterion, it violates the Hessian condition of the refined de Sitter criterion. One can wonder if there is a connection between this violation and our failure to construct such a model with a consistent choice of parameters. In other words, it is possible that the refined de Sitter criterion could be slightly strengthened so that violating the Hessian criterion can still put the theory in the swampland even if the gradient criterion is satisfied. It would also be interesting to investigate whether these sorts of difficulties persist in models that involve other fields, particularly open string moduli [24, 25].

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References

[1] R. D. Peccei and H. R. Quinn, "CP Conservation in the Presence of Instantons," Phys. Rev. Lett. **38**, 1440 (1977).

[2] T. Higaki and F. Takahashi, "Natural and Multi-Natural Inflation in Axion Landscape," JHEP **1407**, 074 (2014)

T. Higaki, and F. Takahashi, "Axion landscape and natural inflation", Physics Letters B 744: 153-159 (2015)

[3] S. Panda, Y. Sumitomo and S. P. Trivedi, "Axions as Quintessence in String Theory," Phys. Rev. D **83**, 083506 (2011) [arXiv:1011.5877 [hep-th]].

[4] P. Svrcek and E. Witten, "Axions In String Theory," JHEP **0606**, 051 (2006) [hep-th/0605206].

[5] C. Vafa "The String Landscape and the Swampland", arXiv:hep-th/0509212

T. D. Brennan, F. Carta, C. Vafa "The String Landscape, the Swampland, and the Missing Corner" arXiv:1711.00864 [hep-th]

G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, “De Sitter Space and the Swampland,” arXiv:1806.08362 [hep-th].

[6] S. K. Garg and C. Krishnan, “Bounds on Slow Roll and the de Sitter Swampland,” arXiv:1807.05193 [hep-th].

[7] H. Ooguri, E. Palti, G. Shiu and C. Vafa, “Distance and de Sitter Conjectures on the Swampland,” arXiv:1810.05506 [hep-th].

[8] D. Andriot, On the de Sitter swampland criterion, Phys. Lett. B 785, 570 (2018) [arXiv:1806.10999 [hep-th]].

A. Hebecker and T. Wrase, The asymptotic dS Swampland Conjecture - a simplified derivation and a potential loophole, arXiv:1810.08182 [hep-th].

G. Dvali and C. Gomez, “On Exclusion of Positive Cosmological Constant,” arXiv:1806.10877 [hep-th].

G. Dvali, C. Gomez and S. Zell, “Quantum Breaking Bound on de Sitter and Swampland,” arXiv:1810.11002 [hep-th].

S. K. Garg, C. Krishnan and M. Zaid, “Bounds on Slow Roll at the Boundary of the Landscape,” arXiv:1810.09406 [hep-th].

H. Fukuda, R. Saito, S. Shirai and M. Yamazaki, “Phenomenological Consequences of the Refined Swampland Conjecture,” arXiv:1810.06532 [hep-th].

[9] H. Murayama, M. Yamazaki and T. T. Yanagida, “Do We Live in the Swampland?,” arXiv:1809.00478 [hep-th].

[10] F. Denef, A. Hebecker and T. Wrase, “de Sitter swampland conjecture and the Higgs potential,” Phys. Rev. D 98, no. 8, 086004 (2018) arXiv:1807.06581 [hep-th].

K. Choi, D. Chway and C. S. Shin, “The dS swampland conjecture with the electroweak symmetry and QCD chiral symmetry breaking,” arXiv:1809.01475 [hep-th].

[11] P. Agrawal, G. Obied, P. J. Steinhardt and C. Vafa, “On the Cosmological Implications of the String Swampland,” Phys. Lett. B **784**, 271 (2018) arXiv:1806.09718 [hep-th].

C. I. Chiang, J. M. Leedom and H. Murayama, “What does Inflation say about Dark Energy given the Swampland Conjectures?,” arXiv:1811.01987 [hep-th].

G. Dvali, C. Gomez and S. Zell, “A Proof of the Axion?,” arXiv:1811.03079 [hep-th].

M. Ibe, M. Yamazaki and T. T. Yanagida, “Quintessence Axion from Swampland Conjectures,” arXiv:1811.04664 [hep-th].

L. Heisenberg, M. Bartelmann, R. Brandenberger and A. Refregier, “Dark Energy in the Swampland,” arXiv:1808.02877 [astro-ph.CO].

Y. Akrami, R. Kallosh, A. Linde and V. Vardanyan, “The landscape, the swampland and the era of precision cosmology,” Fortsch. Phys. **2018**, 1800075 [arXiv:1808.09440 [hep-th]]

A. Ashoorioon, "Rescuing Single Field Inflation from the Swampland," arXiv:1810.04001 [hep-th].

K. Dasgupta, M. Emelin, E. McDonough and R. Tatar, "Quantum Corrections and the de Sitter Swampland Conjecture," arXiv:1808.07498 [hep-th].

[12] S. R. Coleman, "The Uses of Instantons," Subnucl. Ser. **15**, 805 (1979).

[13] J. P. Conlon, "The QCD axion and moduli stabilisation," JHEP **0605**, 078 (2006) [hep-th/0602233].

[14] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, "De Sitter vacua in string theory," Phys. Rev. D **68**, 046005 (2003) [hep-th/0301240].

[15] M. Cicoli, S. De Alwis, A. Maharana, F. Muia and F. Quevedo, "De Sitter vs Quintessence in String Theory," arXiv:1808.08967 [hep-th].

[16] T. C. Bachlechner, K. Eckerle, O. Janssen and M. Kleban, "Multiple-axion framework," Phys. Rev. D **98**, no. 6, 061301 (2018) arXiv:1703.00453 [hep-th].

T. C. Bachlechner, K. Eckerle, O. Janssen and M. Kleban, "Systematics of Aligned Axions," JHEP **1711**, 036 (2017) arXiv:1709.01080 [hep-th].

T. C. Bachlechner, K. Eckerle, O. Janssen and M. Kleban, "Axion Landscape Cosmology," arXiv:1810.02822 [hep-th].

[17] J. J. Blanco-Pillado, C. P. Burgess, J. M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. D. Linde and F. Quevedo, "Racetrack inflation," JHEP **0411**, 063 (2004) [hep-th/0406230].

J. J. Blanco-Pillado, C. P. Burgess, J. M. Cline, C. Escoda, M. Gomez-Reino, R. Kallosh, A. D. Linde and F. Quevedo, "Inflating in a better racetrack," JHEP **0609**, 002 (2006) [hep-th/0603129].

[18] J. Moritz and T. Van Riet, "Racing through the swampland: de Sitter uplift vs weak gravity," JHEP **1809**, 099 (2018) arXiv:1805.00944 [hep-th].

[19] K. Hristov, "Axion Stabilization in Type IIB Flux Compactifications," JHEP **0901**, 046 (2009) arXiv:0810.3329 [hep-th].

[20] J. Moritz, A. Retolaza and A. Westphal, "Toward de Sitter space from ten dimensions," Phys. Rev. D **97**, no. 4, 046010 (2018) arXiv:1707.08678 [hep-th].

[21] P. Agrawal and G. Obied, "Dark Energy and the Refined de Sitter Conjecture," arXiv:1811.00554 [hep-ph].

[22] J. J. Blanco-Pillado, M. A. Urkiola and J. Wachter, "Racetrack Potentials and the de Sitter Swampland Conjectures," arXiv:1811.05463 [hep-th].

[23] Y. Olguin-Trejo, S. L. Parameswaran, G. Tasinato and I. Zavala, "Runaway Quintessence, Out of the Swampland," arXiv:1810.08634 [hep-th].

[24] J. Moritz, A. Retolaza and A. Westphal, "Toward de Sitter space from ten dimensions," Phys. Rev. D **97**, no. 4, 046010 (2018) arXiv:1707.08678 [hep-th].

[25] I. Bena, E. Dudas, M. Graña and S. Lüst, "Uplifting Runaways," arXiv:1809.06861 [hep-th].